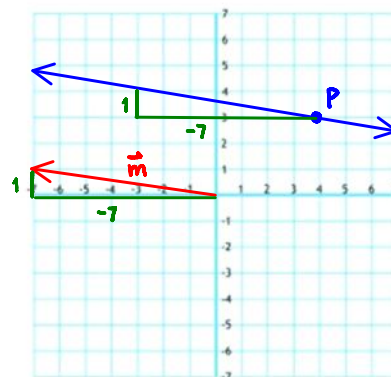


When asked to determine the equation of a line, we tend to use the equation $y = mx + b$. In this lesson we will look at vector and parametric equations which are congruent to the slope-intercept form.

Example:

a) A line passing through $P(4, 3)$ has $\vec{m} = (-7, 1)$ as its direction vector. Sketch this line.



b) A line passes through the points $A\left(\frac{1}{2}, -3\right)$ and $B\left(\frac{3}{4}, \frac{1}{2}\right)$.

Determine a direction vector for this line, and write it using integer components.

$$\vec{AB} = \left(\frac{3}{4} - \frac{1}{2}, \frac{1}{2} - (-3) \right) = \left(\frac{1}{4}, \frac{7}{2} \right)$$

$$\vec{BA} = \left(-\frac{1}{4}, -\frac{7}{2} \right) \quad * \text{infinitely many directions vectors as a solution}$$

→ multiply by LCD to get integer components

$$4 \left(\frac{1}{4}, \frac{7}{2} \right) = (1, 14)$$

* any scalar multiple is also a solution: $t(1, 14)$

Vector and Parametric Equations of a Line in \mathbb{R}^2

Vector Equation: $\vec{r} = \vec{r}_0 + t\vec{m}, t \in \mathbb{R}$

Parametric Equations: $x = x_0 + ta, y = y_0 + tb, t \in \mathbb{R}$

where \vec{r}_0 is the vector from $(0, 0)$ to the point (x_0, y_0) and \vec{m} is a direction vector with components (a, b) .

Example:

a) Determine the vector and parametric equations of a line passing through point $A(1, 4)$ with direction vector $\vec{m} = (-3, 2)$.

b) Determine the coordinates of two points on the line.

c) Are the points $Q(-20, 22)$ and/or $R(-29, 24)$ on the line?

a) vector equation: $\vec{r} = (1, 4) + t(-3, 2), t \in \mathbb{R}$

parametric equations: $x = 1 - 3t, y = 4 + 2t, t \in \mathbb{R}$

b) substitute any number for t

$$t = 2: B(1 - 3(2), 4 + 2(2)) = B(-5, 8)$$

$$t = -5: C(1 - 3(-5), 4 + 2(-5)) = C(16, -6)$$

$$c) \quad 1 - 3t = -20 \quad 4 + 2t = 22$$

$$-3t = -21 \quad 2t = 18$$

$$t = 7 \quad t = 9$$

↑ ≠ ↑
∴ $Q(-20, 22)$ is not on the line

$$1 - 3t = -29 \quad 4 + 2t = 24$$

$$-3t = -30 \quad 2t = 20$$

$$t = 10 \quad t = 10$$

↑ = ↑
∴ $R(-29, 24)$ is on the line

Example:

a) Determine vector and parametric equations for the line containing points $E(-1, 5)$ and $F(6, 11)$.

b) What are the coordinates of the point where this line crosses the x -axis?

a) $\vec{EF} = (6 - (-1), 11 - 5) = (7, 6)$

vector equation: $\vec{r} = (-1, 5) + t(7, 6), t \in \mathbb{R}$ or $\vec{r} = (6, 11) + t(7, 6)$

parametric: $x = -1 + 7t, y = 5 + 6t, t \in \mathbb{R}$ $x = 6 + 7t, y = 11 + 6t$

b) since $y = 0$

$$5 + 6t = 0$$

$$6t = -5$$

$$t = -\frac{5}{6}$$

$$x = -1 + 7\left(-\frac{5}{6}\right)$$

$$= -1 - \frac{35}{6}$$

$$= -\frac{41}{6}$$

$$x\text{-int} : \left(-\frac{41}{6}, 0\right)$$

Example: Determine a vector equation for the line that is perpendicular to $\vec{r} = (4, 1) + s(-3, 2), s \in \mathbb{R}$ and passes through point $P(6, 5)$.

slope of \vec{r} is $-\frac{2}{3}$, perpendicular slope = $\frac{3}{2}$

$$\therefore \vec{r} = (6, 5) + t(2, 3), t \in \mathbb{R}$$

Assignment:

1. A line passes through the points $A(2, 1)$ and $B(-3, 5)$. Write two different vector equations for this line.
2. A line is defined by the parametric equations $x = -2 - t$ and $y = 4 + 2t, t \in \mathbb{R}$.
 - a) Does $R(-9, 18)$ lie on this line?
 - b) Write two vector equations for this line using the given parametric equations.
3. A line passes through the points $M(4, 5)$ and $N(9, 5)$. Determine vector and parametric equations for this line.
4. Write an equation for the line perpendicular to $\vec{r} = (1, -5) + s(3, 5), s \in \mathbb{R}$ passing through $P(2, 0)$.