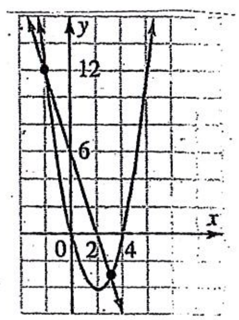
Solving Quadratic Systems

Objective: To solve a quadratic system algebraically

Many quadratic systems can be solved by the same methods that are used to solve linear systems. The methods of SUBSTITUTION and ADDITION as they apply to the solution of quadratic systems are shown in this lesson. When solving a quadratic system, it is usually more convenient to use one method, SUBSTITUTION OR ADDITION, then it is to use the other. If one of the equations is linear, the SUBSTITUTION method is often used. Before finding the solution to a system, think about the graphs of the equations in order to determine the maximum number of solutions.

EXAMPLE 1 Identify the and solve the system $3x + y = 6$
$X^2 = 4x + y$

The system contains the equations of a line and a parabola. The maximum numbers of intersections is two, so there are at most two solutions.



$3x + y = 6$

$y = -3x + 6$ solve the linear equation for y

$x^2 = 4x + y$

$x^2 = 4x + (-3x + 6)$ Substitute $-3x + 6$ for y in the quadratic equation

$x^2 - x - 6 = 0$ solve by factoring

$x = -2$ or $x = 3$

Substitute into the linear equation to find the corresponding values of y

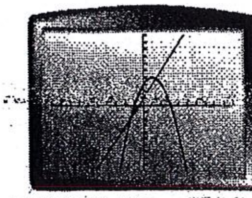
$Y = -3x + 6$	$y = -3x + 6$
$Y = -3(-2) + 6$	$y = -3(3) + 6$
$Y = 12$	$y = -3$

Plug (-2,12) and (3,-3) in both original equations verify the solutions both work!

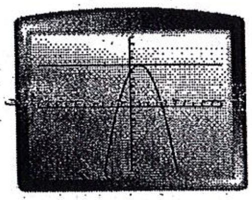
TAKE NOTE: Key Concept Solutions of a linear-quadratic system

A system of one quadratic equation and one linear equation can have two solutions, one solution, or no solutions.

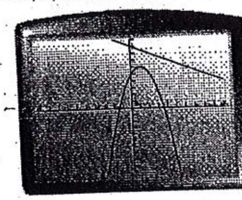
Example: two solutions $y = -x^2 + 2x + 3$
$Y = 2x + 1$



Example: one solution $y = -x^2 + 2x + 5$
$Y = 6$



Example: no solution $y = -x^2 + 2x + 5$
$Y = -\frac{1}{2}x + 9$



Solving a Quadratic System of Equations

What is the solution of the system? $\begin{cases} y = -x^2 - x + 12 \\ y = x^2 + 7x + 12 \end{cases}$

Method 1 Use substitution.

Substitute $y = -x^2 - x + 12$ for y in the second equation. Solve for x.

$-x^2 - x + 12 = x^2 + 7x + 12$	Substitute for y.
$-2x^2 - 8x = 0$	Write in standard form.
$-2x(x + 4) = 0$	Factor.
$x = 0$ or $x = -4$	Solve for x.

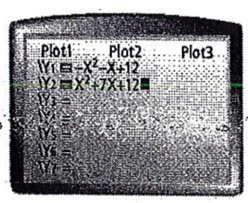
Substitute each value of x into either equation. Solve for y.

$y = x^2 + 7x + 12$	$y = x^2 + 7x + 12$
$y = (0)^2 + 7(0) + 12$	$y = (-4)^2 + 7(-4) + 12$
$y = 0 + 0 + 12 = 12$	$y = 16 - 28 + 12 = 0$

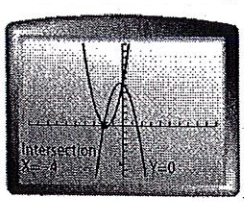
The solutions are $(0, 12)$ and $(-4, 0)$.

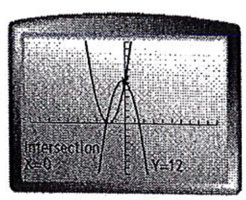
Method 2 Graph the equations.

Use a graphing calculator. Define functions $\mathbf{Y_1}$ and $\mathbf{Y_2}$.



Use the **INTERSECT** feature to find the points of intersection.





The solutions are $(-4, 0)$ and $(0, 12)$.

#'s 1-6 ; 13-18

PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

For additional support when completing your homework, go to PearsonTEXAS.com.

Solve each system by ~~graphing~~ algebraically. Check your answers.

1. $\begin{cases} y = -x^2 + 2x + 1 \\ y = 2x + 1 \end{cases}$
2. $\begin{cases} y = x^2 - 2x + 1 \\ y = 2x + 1 \end{cases}$
3. $\begin{cases} y = x^2 - x + 3 \\ y = -2x + 5 \end{cases}$
4. $\begin{cases} y = 2x^2 + 3x + 1 \\ y = -2x + 1 \end{cases}$
5. $\begin{cases} y = -x^2 - 3x + 2 \\ y = x + 6 \end{cases}$
6. $\begin{cases} y = -x^2 - 2x - 2 \\ y = x - 4 \end{cases}$

Solve each system by substitution. Check your answers.

7. $\begin{cases} y = x^2 + 4x + 1 \\ y = x + 1 \end{cases}$
8. $\begin{cases} y = -x^2 + 2x + 10 \\ y = x + 4 \end{cases}$
9. $\begin{cases} y = -x^2 + x - 1 \\ y = -x - 1 \end{cases}$
10. $\begin{cases} y = 2x^2 - 3x - 1 \\ y = x - 3 \end{cases}$
11. $\begin{cases} y = x^2 - 3x - 20 \\ y = -x - 5 \end{cases}$
12. $\begin{cases} y = -x^2 - 5x - 1 \\ y = x + 2 \end{cases}$

Solve each system. algebraically

13. $\begin{cases} y = x^2 + 5x + 1 \\ y = x^2 + 2x + 1 \end{cases}$
14. $\begin{cases} y = x^2 - 2x - 1 \\ y = -x^2 - 2x - 1 \end{cases}$
15. $\begin{cases} y = -x^2 - 3x - 2 \\ y = x^2 + 3x + 2 \end{cases}$
16. $\begin{cases} y = -x^2 - x - 3 \\ y = 2x^2 - 2x - 3 \end{cases}$
17. $\begin{cases} y = -3x^2 - x + 2 \\ y = x^2 + 2x + 1 \end{cases}$
18. $\begin{cases} y = x^2 + 2x + 1 \\ y = x^2 + 2x - 1 \end{cases}$

19. **Apply Mathematics (1)(A)** A manufacturer is making cardboard boxes by cutting out four equal squares from the corners of a rectangular piece of cardboard and then folding the remaining part into a box. The length of the cardboard piece is 1 in. longer than its width. The manufacturer can cut out either 3×3 in. squares, or 4×4 in. squares. Find the dimensions of the cardboard for which the volume of the boxes produced by both methods will be the same.

20. **Justify Mathematical Arguments (1)(G)** Can you solve the system of equations shown by graphing? Justify your answer. Can you solve this system using another method? If so, solve the system and explain why you chose that method. $\begin{cases} x = y^2 + 2y + 1 \\ y = x - 4 \end{cases}$

Solve each system by substitution.

21. $\begin{cases} x + y = 3 \\ y = x^2 - 8x - 9 \end{cases}$
22. $\begin{cases} y - 2x = x + 5 \\ y + 1 = x^2 + 5x + 3 \end{cases}$
23. $\begin{cases} y - \frac{1}{2}x^2 = 1 + 3x \\ y + \frac{1}{2}x^2 = x \end{cases}$
24. $\begin{cases} x + y - 2 = 0 \\ x^2 + y - 8 = 0 \end{cases}$
25. $\begin{cases} x^2 - y = x + 4 \\ x - 1 = y + 3 \end{cases}$
26. $\begin{cases} 2y = y - x^2 + 1 \\ y = x^2 - 5x - 2 \end{cases}$

27. **Evaluate Reasonableness (1)(B)** Your friend wants to solve the system $y = x^2$ and $y = x$. She concludes that the solution is (0, 0), because $0 = 0^2$. What would you tell your friend about her solution?

28. **Create Representations to Communicate Mathematical Ideas (1)(E)** A circle with radius of 5 and center at (0, 0) and a line with slope -1 and y-intercept 7 are graphed on a coordinate plane. What are the solutions to the system of equations?

Quadratic-Quadratic systems of equations

#'s 1-6

Short Answer

What is the solution of the quadratic system of equations?

1. $\begin{cases} y = x^2 - 2x + 4 \\ y = -x^2 - 2x + 4 \end{cases}$

2. $\begin{cases} y = x^2 + 3x + 2 \\ y = x^2 + 5x - 4 \end{cases}$

3. $\begin{cases} y = x^2 + 18x + 35 \\ y = -x^2 + 2x + 5 \end{cases}$

4. $\begin{cases} y = x^2 + 16x + 32 \\ y = -x^2 + 2 \end{cases}$

Solve the system algebraically.

5. $\begin{cases} y = x^2 + 3x - 7 \\ x + y = -2 \end{cases}$

Use graphing to find the solutions to the system of equations. Then solve the system algebraically. Be sure to show work for finding the vertex!!

6. $\begin{cases} y = -x^2 - 2x + 3 \\ x - y = 1 \end{cases}$

Solve each quadratic equation by any method.

7. $x^2 - 12x + 32 = 0$

8. $49x^2 - 25 = 0$

9. $x^2 - 8x + 16 = 16$

Example 1

$3x+y=6 \rightarrow y=-3x+6$

$x^2=4x+y \rightarrow x^2-4x=y$

$x^2-4x=-3x+6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3 \quad x=-2$

For Both equations when $x=3$, $y=-3$

when $x=-2$ $y=12$ for Both equations

$(-2,12)\ (3,-3)$ Solution

Reverse side

Example 2 Solutions

$y=-x^2+2x+3$

$y=2x+1$

$-x^2+2x+3=2x+1$

$-x^2+2=0$

$x^2=2 \quad x=\pm\sqrt{2}$

$y=-(\sqrt{2})^2+2(\sqrt{2})+3$

$y=1+2\sqrt{2}$

$y=-(-\sqrt{2})^2+2(-\sqrt{2})+3$

$y=1-2\sqrt{2}$

$y=2(\sqrt{2})+1$

$y=2(-\sqrt{2})+1$

$y=1-2\sqrt{2}$

Solutions

$\sqrt{2},\ 1+2\sqrt{2}$

$-\sqrt{2},\ 1-2\sqrt{2}$

Example 1 Solution

$y=-x^2+2x+5$

$y=6$

$6=-x^2+2x+5$

$-x^2+2x-1=0$

$-(x^2-2x+1)=0$

$x^2-2x+1=0$

$x-1 \quad x-1=0$

$x=1 \quad x=1$

Double Root

Solutions

$x=1$

$y=-(1)^2+2(1)+5$

$y=6$

Example No solution

$y=-x^2+2x+5$

$y=-\frac{1}{2}x+9$

$-x^2+2x+5=-\frac{1}{2}x+9$

$\frac{1}{2}x^2-2x+4=0$

$x=\dfrac{2\pm\sqrt{4-4(2\quad)}}{1}$

$x=\dfrac{2\pm 2i}{1}$ No Solution

Quadratic Systems

$-x^2 - x + 12 = x^2 + 7x + 12$

$2x^2 + 8x = 0$

$2x(x+4) = 0$

$x = 0 \quad x = -4$

when $x = 0$

$-(0)^2 - 0 + 12 = 12$

$(0)^2 + 7(0) + 12 = 12$

when $x=0$

$-(0)^2 - 0 + 12 = 12$ $\quad 0, 12$

$(0)^2 + 7(0) + 12 = 12$

when $x = -4$

$-(-4)^2 - (-4) + 12 = 0$

$(-4)^2 + 7(-4) + 12 = 0$

solutions $0, 12 \quad -4, 0$

Practice and Application Exercises

Practice and application Exercises

#1–6

1) $-x^2+2x+1=2x+1$

$-x^2=0 \quad x=0$

when $x=0$

$-(0)^2+2(0)+1=1$

$2(0)+1=1$

solution $0,1$

2.) $x^2-2x+1=2x+1$

$x^2-4x=0$

$x(x-4)=0$

$x=0 \quad x=4$

when $x=0$

$(0)^2-2(0)+1=1$

$2(0)+1=1$

when $x=4$

$(4)^2-2(4)+1=9$

$2(4)+1=9$

solution

$(0,1)\ (4,9)$

3.) $x^2-x+3=-2x+5$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \quad x=1$

when $x=-2$

$(-2)^2-(-2)+3=9$

$-2(-2)+5=9$

when $x=1$

$(1)^2-1+3=3$

$-2(1)+5=3$

solution

$(-2,9)(1,3)$

4) $2x^2+3x+1=-2x+1$

$2x^2+5x=0$

$x(2x+5)=6$

$x=0 \quad x=-5/2$

when $x=-5/2$

$2(-5/2)^2+3(-5/2)+1$

$\frac{25}{2}-\frac{15}{2}+\frac{2}{2}=6$

$-2\left(\frac{-5}{2}\right)+1=6$

when $x=0$

$0^2+0+1=1$

$-2(0)+1=1$

solution

$0,1 \quad \frac{-5}{2},6$

5.) $-x^2-3x+2=x+6$

$-x^2-4x-4=0$

$-(x^2+4x+4)=0$

$-(x+2)^2=0$

$x=-2$

when $x=-2$

$-(-2)^2-3(-2)+2=4$

$-2+6=4$

solution $(-2,4)$

6.) $-x^2-2x-2=x-4$

$-x^2-3x+2=0$

$-(x^2+3x-2)=0$

$x=\frac{-3+\sqrt{17}}{2}$

$x=\frac{-3-\sqrt{17}}{2}$

Solution

$\frac{-3+\sqrt{17}}{2},\frac{-11+\sqrt{17}}{2}$

$\frac{-3-\sqrt{17}}{2},\frac{-11-\sqrt{17}}{2}$

When $x=\frac{-3+\sqrt{17}}{2}$

$-\left(\frac{-3+\sqrt{17}}{2}\right)^2-2\left(\frac{-3+\sqrt{17}}{2}\right)-2=\frac{-11+\sqrt{17}}{2}$

$\frac{-3+\sqrt{17}}{2}-4=\frac{-11+\sqrt{17}}{2}$

when $x=\frac{-3-\sqrt{17}}{2}$

$-\left(\frac{-3-\sqrt{17}}{2}\right)^2-2\left(\frac{-3-\sqrt{17}}{2}\right)-2=\frac{-11-\sqrt{17}}{2}$

$\frac{-3-\sqrt{17}}{2}-\frac{8}{2}=\frac{-11-\sqrt{17}}{2}$

Practice and Application Exercises

3)
$y = x^2 + 5x + 1$
$y = x^2 + 2x + 1$

$x^2 + 5x + 1 = x^2 + 2x + 1$
$3x = 0$
$x = 0$

$(0)^2 + 5(0) + 1 = 1$
$(0)^2 + 2(0) + 1 = 1$

Solution 0, 1

4)
$y = x^2 - 2x - 1$
$y = -x^2 - 2x - 1$

$x^2 - 2x - 1 = -x^2 - 2x - 1$
$2x^2 = 0$
$x = 0$

$(0)^2 - 2(0) - 1 = -1$
$-(0)^2 - 2(0) - 1 = -1$

Solution 0, -1

5
$y = -x^2 - 3x - 2$
$y = x^2 + 3x + 2$

$2x^2 + 6x + 4 = 0$
$2(x^2 + 3x + 2) = 0$
$2(x+2)(x+1) = 0$
$x = -2 \quad x = -1$

Solution -2, 0 -1, 0

when $x = -2$
$-(-2)^2 - 3(-2) - 2 = 0$
$(-2)^2 + 3(-2) + 2 = 0$

when $x = -1$
$-(-1)^2 - 3(-1) - 2 = 0$
$(-1)^2 + 3(-1) + 2 = 0$

16.) $y = -x - x - 3$

$y = 2x^2 - 2x - 3$

$2x - 2x - 3 = -x - x - 3$

$3x^2 - x = 0$

$3x(x - \frac{1}{3}) = 0 \quad x = 0 \quad x = 1/3$

when $x = 0$

$-(0)^2 - (0) - 3 = -3$

$2(0)^2 - 2(0) - 3 = -3$

when $x = 1/3$

$-(\frac{1}{3})^2 - \frac{1}{3} - 3 = \frac{-31}{9}$

$2(\frac{1}{3})^2 - 2(\frac{1}{3}) - 3 = \frac{-31}{9}$

Solution

$0, -3 \quad \frac{1}{3}, \frac{-31}{9}$

17.) $y = -3x^2 - x + 2$

$y = x^2 + 2x + 1$

$4x^2 + 3x - 1 = 0$

$(x+1)(4x-1) = 0$

$x = -1 \quad x = 1/4$

Solution

$-1, 0 \quad \frac{1}{4}, \frac{25}{16}$

when $x = 1/4$

$-3(\frac{1}{4})^2 - \frac{1}{4} + \frac{8}{4} = \frac{25}{16}$

$(\frac{1}{4})^2 + 2(\frac{1}{4}) + \frac{4}{4} = \frac{25}{16}$

when $x = -1$

$-3(-1)^2 - (-1) + 2 = 0$

$(-1)^2 + 2(-1) + 1 = 0$

18.) $y = x^2 + 2x + 1$

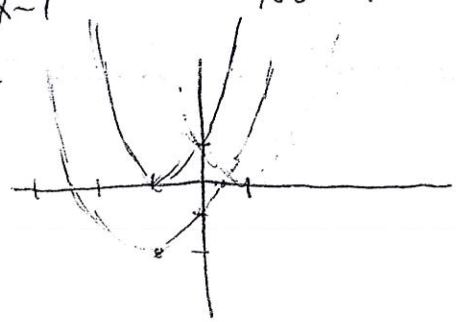
$y = x^2 + 2x - 1$

$y = (x+1)^2 - 2$

$-1 \pm \sqrt{2}$

$0 = 2$

NO intersection



#1-6 Quadratic Quadratic S.O.S.

1) $x^2 - 2x + 4 = -x^2 - 2x + 4$

$2x^2 = 0$

$x = 0$

$(0)^2 - 2(0) + 4 = 4$

$-(0)^2 - 2(0) + 4 = 4$

Solution 0, 4

2) $x^2 + 3x + 2 = x^2 + 5x - 4$

$0 = 2x - 6$

$x = 3$

$(3)^2 + 3(3) + 2 = 20$

$(3)^2 + 5(3) - 4 = 20$

Solution 3, 20

3) $x^2 + 18x + 35 = -x^2 + 2x + 5$

$2x^2 + 16x + 30 = 0$

$2(x^2 + 8x + 15) = 0$

$2(x+5)(x+3) = 0$

$x = -5 \quad x = -3$

Solution

$-3, -10 \quad -5, -30$

when $x = -5$

$(-5)^2 + 18(-5) + 35 = -30$

$-(-5)^2 + 2(-5) + 5 = -30$

when $x = -3$

$(-3)^2 + 18(-3) + 35 = -10$

$-(-3)^2 + 2(-3) + 5 = -10$

4) $x^2 + 16x + 32 = -x^2 + 2$

$2x^2 + 16x + 30 = 0$

$2(x+5)(x+3) = 0$

$x = -5 \quad x = -3$

Solution

$(-5, -23)(-3, -7)$

when $x = -5$

$(-5)^2 + 16(-5) + 32 = -23$

$-(-5)^2 + 2 = -23$

when $x = -3$

$(-3)^2 + 16(-3) + 32 = -7$

$-(-3)^2 + 2 = -7$

5) $y = -x^2 - 2x + 3$

$y = x - 1$

when $x = -4$

$-(-4)^2 - 2(-4) + 3 = -5$

$y = -4 - 1 = -5$

$-x^2 - 3x + 4 = 0$

$-(x^2 + 3x - 4) = 0$

$-(x+4)(x-1)$

$x = -4 \quad x = 1$

when $x = 1$

$-(1)^2 - 2(1) + 3 = 0$

$y = 1 - 1 = 0$

Solution

$(-4, -5)(1, 0)$

(5)

$y = +x^2 + 3x - 7$

$y = -x - 2$

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5 \quad x = 1$

when $x = -5$

$(-5)^2 + 3(-5) - 7 = 3$

$y = -(-5) - 2 = 3$

when $x = 1$

$(1)^2 + 3(1) - 7 = -3$

$y = -(1) - 2 = -3$

Solution $-5, 3 \quad 1, -3$

⑤ $y = +x^2 + 3x - 7$

$y = -x - 2$

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5 \quad x = 1$

when $x = -5$

$(-5)^2 + 3(-5) - 7 = 3$

$y = -(-5) - 2 = 3$

when $x = 1$

$(1)^2 + 3(1) - 7 = -3$

$y = -(1) - 2 = -3$

Solution $-5, 3 \quad 1, -3$